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<http://panm20.math.cas.cz>
panm@math.cas.cz



ABSTRAKTY

Dynamics of the artificial slope

Stanislav Bartoň

The slope shape is replaced by a 3D regression function, which corresponds with high precision to the position of several hundred points, which were determined with high precision on the surface of the slope body. The position of several points was repeatedly measured for several years. The time changes in the position of these points were used to create regression functions that describe vertical movements - slope settlement and horizontal movements - slope movement. The model results are presented in the form of mathematical relationships and visualized in the program Maple environment.

Goal-oriented a posteriori error estimates for elliptic problems discretized by the DG method

Ondřej Bartoš, Vít Dolejší

We deal with the numerical solution of linear elliptic problems using the discontinuous Galerkin method with focus on the goal-oriented a posteriori error estimates. Our aim is to estimate the error of the quantity of interest represented by a linear functional. The abstract error estimate is based on the knowledge of the (exact) solution z of the dual problem corresponding to the primal one. In order to define a computable estimate, the dual solution z has to be replaced by its approximation z_h and the remaining term represented by difference $z - z_h$ is usually neglected. In this presentation we propose an approach which estimates the neglected term. Consequently, we are able to derive an upper error estimate. These types of estimates are known for the conforming finite element approximation but are new in the framework of discontinuous Galerkin method. Moreover, we introduce an efficient implementation approach allowing a simultaneous solution of the primal and dual problems. Finally, numerical examples will be presented.

Numerical methods for hydro-mechanics in standard and disturbed continua

Radim Blaheta

Mathematical modelling of coupled processes (multiphysics) became important for many applications. Besides others, the coupling of flow in porous material with mechanical deformation is the problem with a lot of applications in geomechanics and biomechanics. Numerical realization of the hydro-mechanical models can use different finite element techniques for space discretization and an unconditionally stable discretization in time. Arising systems can be solved by iterative methods which utilize the block structure given by the individual processes. These iterations can be also viewed as a way of coupling of the physical processes and a scheme which is suitable for different applications. One specific application is hydro-mechanics in disturbed continua with the meaning of continua with disturbed zones or fractures, which frequently appear in the analysis of processes in a geological environment with disturbed brittle rocks. The lecture is partly based on joint work with O. Axelsson, M. Béréš, S. Domesová, D. Horák, J. Kružík and T. Luben.

Numerické modelování nestlačitelného vazkého proudění na bázi isogeometrické analýzy I

Marek Brandner

Přednáška bude zaměřena na některé aktuální problémy související s využitím isogeometrické analýzy pro numerickou simulaci vazkého nestlačitelného proudění. V první části zmíníme formulaci problému, diskrétní formulaci, volbu sítí a volbu bazových funkcí, lineární a nelineární stabilizační techniky a základní informace o linearizaci problému a řešení získaných soustav lineárních algebraických rovnic.

MLMC for Fractured Porous Media: a multiscale approach

Jan Březina

The multilevel Monte Carlo method can lead to a computationally efficient mean estimating algorithm for the PDE based random variables. In particular, this has already been demonstrated for different kind of elliptic and parabolic PDE with uncertain data. We shall investigate a numerical interpolation between DFN and a continuum model for a Darcy flow in a fractured media with uncertain data. This interpolation then allows us to construct the MLMC estimator. We will present numerical results for the 2d case and discuss the technical issues of the 3d case. Finally, we will briefly present the MLMC library for Python.

Wavelet method for option pricing under the two-asset Merton jump-diffusion model

Dana Černá

We focus on the pricing of two-asset European options under the Merton model, which is represented by a nonstationary integro-differential equation with two state variables. The Crank-Nicolson scheme for time discretization combined with a wavelet-based method for spatial discretization is used for its numerical solution. The drawback of most classical methods is that matrices arising from discretization are full. Compared to that, the wavelet method enables the approximation of discretization matrices with quasi-sparse matrices. This is important for an efficient application of iterative methods for the solution of the resulting systems of linear algebraic equations as well as for the efficient computation of matrices arising from discretization of an integral term. Another advantage is that the diagonally preconditioned discretization matrices have uniformly bounded condition numbers. To illustrate the efficiency and applicability of the method, we provide numerical experiments for a European option on the maximum of two assets.

Numerické modelování nestlačitelného vazkého proudění na bázi isogeometrické analýzy II

Jiří Egermaier

Přednáška bude zaměřena na některé aktuální problémy související s využitím isogeometrické analýzy pro numerickou simulaci vazkého nestlačitelného proudění. V této druhé části budou diskutovány otázky související s předpokládáním pro získané soustavy lineárních algebraických rovnic. Dále srovnáme efektivitu jednotlivých iteračních technik s ohledem na stupeň bazových funkcí. Na závěr budou prezentovány výsledky na několika úlohách (zpětný schod, profil, kavita).

Numerical solution of a stochastic model of a ball-type vibration absorber

Cyril Fischer

The mathematical model of a ball-type vibration absorber, possibly connected to an elastic slender structure, represents a non-linear differential system which includes non-holonomic constraints. When a random ambient excitation is taken into account the system has to be treated as a stochastic differential equation. Depending on the level of simplification, an analytical solution is not practicable and numerical solution procedures have to be applied. The contribution presents a feasible way to numerically obtain stochastic properties of a vibration absorber's behaviour.

Multilevel BDDC for incompressible flow

Martin Hanek

We apply Multilevel Balancing Domain Decomposition based on Constraints (BDDC) to the stationary incompressible Navier-Stokes equations discretized by the finite element method. The algorithm is applied to saddle-point nonsymmetric linear systems obtained by Picard's linearisation using Taylor-Hood finite elements. The linear system is solved by the BiCGstab method using one step of Multilevel BDDC as the preconditioner. Numerical results for the benchmark problem of 3-D lid driven cavity are presented.

Hybrid LSQR regularization for inverse problems in Single Particle Analysis

Eva Havelková, Iveta Hnětynková

In this contribution we concentrate on discrete inverse problems $Ax = b$ arising in cryo-electron microscopy single particle analysis. Since these problems have specific properties (such as high sensitivity of the solution on noise present in the observation b , large noise level, etc.), their numerical solution is highly challenging. We describe a variant of iterative hybrid LSQR method with inner Tikhonov regularization and show its effectivity on these problems. Special attention is given to parameter-choice method for the inner Tikhonov regularization as well as suitable stopping criterion for the outer LSQR iterations. Since the underlying application belongs to the category of large scale problems, issues related to effective implementation are also studied. Based on numerical experiments, we examine the behavior of the proposed method on synthetic and real datasets. Finally, several open questions are formulated.

Regularization properties of Krylov subspace projections

Iveta Hnětynková

Krylov subspace methods represent powerful iterative regularization tools for large-scale linear ill-posed problems contaminated by unknown noise. Regularization is here achieved via projection on a sequence of nested Krylov subspaces dominated by low-frequency vectors. Depending on the particular data, the projected subproblems eventually inherit the ill-posed nature of the original problem. Thus the methods exhibit semiconvergence, where the number of iterations plays the role of the regularization parameter. In this talk we present analytical results leading to understanding of regularization properties of selected Krylov subspace methods and discuss various ways to overcome the semiconvergence phenomenon. Influence of finite precision will be also considered.

Curve reconstruction from a set of measured points

Marta Hlavová

In this article, a method of cubic spline curve fitting to a set of points passing at the prescribed distance from the input points obtained by measurement on a coordinate measuring machine is described. When reconstructing the shape of the measured object from the points obtained by real measurements, it is always necessary to consider the measurement uncertainty (tenths to tens of micrometers). This uncertainty is not zero, therefore interpolation methods where the resulting curve passes through the given points do not lead to acceptable results in practice. Also, conventional B-spline approximation methods cannot be used because, for real distances between the measured points (tenths to units of millimeters), the distance of the input data from the resulting approximation curve is much greater than the measurement uncertainty considered. The proposed reconstruction method allows to control the maximum distance of the resulting curve from the input data and thus to respect the uncertainty with which the input data was obtained.

Identifikace parametrů obecného Kelvinova řetězce z křivky součinitele dotvarování pomocí optimalizace hejnem částic

F. Hokeš, M. Trcala, J. Kala, I. Němec

Konstitutivní vztah v numerickém řešení vlivu dotvarování při výpočtu odezvy stavebních konstrukcí lze definovat pomocí Kelvinova řetězce, z něhož odvozená funkce pro dotvarování při konstantním napětí lze vyjádřit Dirichletovou řadou aproximující normovou křivku součinitele dotvarování. Při praktickém použití materiálových modelů založených na visko-elastických řetězcích vzniká problém, jak předem určit koeficienty řady. S ohledem na aktuální výkon počítačů lze pro identifikaci parametrů použít výpočetní hrubou sílu ve formě optimalizace hejnem částic, která zpřesňuje obvykle používaný odhad metodou nejmenších čtverců. Předkládaný článek prezentuje použití a výsledky této identifikační metody na praktickém příkladu dotvarování tláčeného sloupu.

DG method for valuation of European options under the variance Gamma process

Jiří Hozman, Tomáš Tichý

We present the discontinuous Galerkin method applied to valuation of European options assuming that the underlying follows the variance Gamma (VG) process. This pure jump process has an infinite number of jumps in any interval of time and thus the corresponding VG model was proposed as an extension of geometric Brownian motion to overcome some of the limitations of the Black-Scholes approach. The evolution of the option prices under this model can be expressed in the form of a partial integro-differential equation, which involves both integrals and derivatives of an unknown option value function. With a localization to a bounded spatial domain, the pricing equation is discretized by the discontinuous Galerkin method over a finite element mesh and it is integrated in temporal variable by a semi-implicit Euler scheme. The special attention is paid to the proper discretization of jump components, which is based on splitting integrals with respect to the domain according to the size of the jumps. Finally, the preliminary numerical results demonstrate the capability of the numerical scheme presented within the reference benchmarks.

Posouzení nejistého chování modelů dotvarování a smrštění betonu

Jan Chleboun

U betonových staveb a konstrukcí se často předpokládá životnost 100 let, přitom však nepanuje shoda v tom, jak tak dlouhodobé chování betonu matematicky modelovat, a ani neexistuje uspokojivý počet vhodných měření, o něž by se modelování závažných procesů dotvarování a smrštění betonu mohlo opřít. V současnosti existuje několik (4-5) mezinárodně uznávaných, leč částečně odlišných modelů používaných při výpočtech. Cílem příspěvku je metodami fuzzy množin a Dempsterovy-Shaferovy teorie posoudit úroveň shody výstupů poskytovaných jednotlivými modely.

Optimal control of the parallel system with continuous wear

Čeněk Jirsák

Maintenance optimization is a common topic in mathematical reliability theory. Motivation for our model are continuously deteriorating systems consisting of components working in parallel with a redundancy. An example of such a system might be a group of coal mills in a power plant, where 7 out of 8 mills must be operating for an efficient coal burning. Such systems are usually modeled as multistate stochastic systems using standard tools (mainly Markov chains and processes). Our focus is to apply continuous deterioration to the model. So far we can find an optimal policy only for a linear deterministic models. For more general settings we use numerical approximation. The numerical approach for finding an optimal policy will be the focus of the poster (talk).

Výpočet detekčních vlastností v binárním symetrickém kanálu

Štěpán Klapka, Adam Rychtář

V posteru jsou prezentovány výsledky prohledávání vhodných generujících polynomů mezi polynomy 32. stupně. Cílem bylo nalezení cyklických kódů s lepšími detekčními vlastnostmi než v praxi doposud používaných, například v protokolu Ethernetu. Měřítkem detekčních vlastností je uvažována pravděpodobnost nedetekovatelné chyby v modelu BSC. Testovány byly nerozložitelné samoadjugované generující polynomy, které zaručují minimální váhu rovnou pěti pro slova délky do 65536 bitů. Analýzou těchto generujících polynomů bylo zjištěno, že žádný z kódů nesplňuje podmínku správnosti. Na druhou stranu, podařilo se najít takové generující polynomy, jejichž průběh pravděpodobnosti nedetekovatelné chyby jimi generovaných kódů je příhodnější než u kódů používaných v technické bezpečnosti nebo v přenosových zařízeních.

RTIN Based Meshes in 2D and 3D with applications to geotechnical stability analysis

Alexej Kolcun, Stanislav Sysala

The local refinement of the mesh, based on the longest-side partition of rectangular isosceles triangle leads to Rigt-Triangulated Irregular Network (RTIN) meshes. We describe a 3D extension of this strategy based on the balanced structure of the octal tree. The suggested local mesh refinement is combined with an adaptive strategy and illustrated on a model geotechnical stability problems. These problems are based on perfect plasticity and related limit load analysis. The contribution is a joint work with Stanislav Sysala.

Explicit time integration in finite element method for structural dynamic, wave propagation and contact-impact problems: a recent progress

Radek Kolman

In the contribution, recent progress in explicit time integration for finite element analysis is presented with attention on modern and advanced methodologies including the direct mass inversion via localized Lagrange multipliers, mass scaling, estimation of stable time step size, enforcing nontrivial Dirichlet boundary conditions, heterogeneous explicit scheme with local stepping for the elimination of spurious oscillations, etc. In the contact-impact problems, the partitioned bipenalty method for enforcing of contact constraints is discussed, where this method does not attack the stability limit against the classical penalty method. Several benchmark tests and applications in structural dynamic, wave propagation and impact-contact problems are presented.

Active Set Expansion Strategies in MPRGP Algorithm

Jakub Kruzik, David Horak, Martin Cermak, Lukas Pospisil,
Marek Pecha

The standard MPRGP expansion uses a projected line search in the free gradient direction with a fixed step length. Such a scheme is often too slow to identify the active set, requiring a large number of expansions. We propose to use adaptive step lengths based on the current gradient, which guarantees the decrease of the unconstrained cost function with different gradient-based search directions. Moreover, we also propose expanding the active set by projecting the optimal step for the unconstrained minimization. Numerical experiments demonstrate the benefits (up to 78 pct decrease in the number of Hessian multiplications) of our expansion step modifications on two benchmarks – contact problem of linear elasticity solved by TFETI and machine learning problems of SVM type, both implemented in PERMON toolbox.

Low-Mach consistency of a class of linearly implicit schemes for the compressible Euler equations

Václav Kučera

In 2004, while attending his first conference - PANM 12, V. Kučera presented results from his Master thesis under the supervision of M. Feistauer - a semi-implicit discontinuous Galerkin scheme for the compressible Euler equations. The scheme worked surprisingly well for very low Mach number flows, as demonstrated by numerical experiments. Sixteen years later, we finally know why. We will present the corresponding analysis of the asymptotic preserving properties of the semi-implicit linearization in time. This is performed for a larger class of linearly implicit schemes including e.g. the so-called RS-IMEX scheme.

Computational issues and challenges in soil system science

Michal Kuráž

Mathematical modeling in soil system sciences typically covers three distinct fields: hydrodynamics, thermodynamics and transport of solutes. A very typical property of all hydrodynamical processes in soils is an extremely low velocity of the liquid motion, flow regime is typically laminar. Due to low kinetic forces the flow field is significantly affected by osmotic and temperature gradient. Further, the flow area is scattered into microscopic pore paths, where phase changes such as evapotranspiration and ice crystallization can significantly affect local porous medium hydraulic properties. Setting up boundary conditions representing meteorological conditions is again nontrivial. Rainfall intensity can often exceed infiltration capacity of the porous medium, and then surface runoff with erosion forces generates. In this talk mathematical concepts for solving these coupled hydro/thermo dynamic systems will be presented together with different computational techniques for solving these equations efficiently.

Mathematical modeling of laser welding

Josef Bradáč, Jiří Hozman, Jan Lamač

The weldability of iron aluminide alloys is very difficult when using conventional welding methods. Hence, the numerical analysis of the welding process can help to improve its efficiency. In this contribution we model the temperature distribution during the laser welding by the nonlinear heat conduction equation equipped with the boundary conditions describing the convection and radiation heat losses. The radiated energy is represented by Stefan-Boltzmann's law whereas Newton's law of cooling was used to include the convection heat loss. For the numerical solution of arising nonlinear PDE we applied the discontinuous Galerkin method.

Analýza povrchu ploch s aplikací na pánevní kosti

Nikola Pajerová, Ivana Linkeová

Velmi často řešený problém v počítačové grafice je analýza tvaru dané plochy a s tím spojené určování podobnosti či rozdílů mezi dvěma a více plochami. Často se k řešení podobnosti dvou ploch využívá tvarové rozdělení, tedy statistická metoda, která porovnává pravděpodobnostní rozdělení získané z dané tvarové funkce. V tomto článku se budeme zabývat porovnáváním tvarových funkcí (tj. funkcí, které měří jednoduché geometrické charakteristiky – vzdálenosti, plochy, úhly atp.) při použití na ploškách pánevních kostí (tzv. symfýzy) různě starých jedinců. Veškeré objekty byly naskenovány optickým skenerem a uloženy ve formátu trojúhelníkové sítě. Ukážeme si, jak dané tvarové funkce vypadají na objektu složeném ze základních geometrických těles a jaké výsledky ukazují na daných kostech.

Systems biology analysis of a drug metabolism and xenobiotic metabolizing enzyme induction process (with special attention to the slow-fast decomposition)

Stepan Papacek, Volodymyr Lynnyk, Branislav Rehak

The common goal of systems biology and systems pharmacology (i.e., systems biology applied to the field of pharmacology) is to rely less on trial and error in designing therapeutic schedules. In this contribution we present, on the paradigmatic example of rifampicin metabolism published by Luke et al. (2010), new dimensionless formulation of this compartmental model in form of an ODE system. Our aim is to algorithmically develop an enhanced (yet biologically meaningful) model, which can be used for further analysis and optimization of drug delivery. We pay particular attention to the slow-fast decomposition enabling eventually an ODE system order reduction. For both full and reduced model we calculated the functional form of hepatic enzyme induction, i.e., time course of CYP3A4. The comparison of the full system behavior with the reduced model is presented and future prospects are proposed.

Adaptive multilevel solvers with p-robust behavior

Jan Papež

We present a multilevel solver which reduces the error in each iteration by a factor that can be bounded by a constant independent on the polynomial degree of the FEM discretization of an underlying second-order elliptic diffusion problem. Multilevel methods are among the most computationally efficient algebraic solvers, however, they often require careful choice of smoothers on each level and of the number of smoothing steps. For our solver, we present two techniques how to make the smoother fully adaptive. This is done by using a posteriori error estimator to select levels and/or regions for an additional smoothing. Such techniques can further reduce the overall computational cost of the solver while the p-robust convergence can still be proven. We present numerical tests confirming the p-robust behavior of the solver with the adaptive smoother.

This is a joint work with Ani Miraçi and Martin Vohralík (Inria Paris).

Spectral/hp elements in fluid structure interaction

Jan Pech

The work presents simulations of incompressible fluid flow interacting with a moving rigid body. Numerical algorithm for incompressible Navier-Stokes equations is applied to two types of body motion, prescribed and flow-induced. Used approximation in spatial coordinates is based on spectral/hp element method what allows construction of coarser computational meshes. Relative deformation of elements due to the body motion is then smaller and brings different limits for stability. Results show performance of our 2D solver for various geometries from simple cylinder to aerodynamic profiles with emphasize to differences which are introduced using the high-order method.

Supper-time-stepping for traffic flow problem

Jan Příkryl

In 1996, Alexiades and colleagues published an interesting paper on super-time-stepping method for solving PDEs with explicit time-stepping schemes. Their method achieves interesting speedups by allowing the stability requirement to be met only at every N -th step, and by "wisely choosing the intermediate steps". The method has been demonstrated to work well for parabolic PDEs. Inspired by our interest in traffic modelling, we present an extension to this approach for traffic flow problem and analyse the effect of super-time-stepping when modelling the highway flow using the Aw-Rascle-Zhang model.

Stochastic Galerkin method

Ivana Pultarová

For numerical solution of differential equations with parameters one can use Monte Carlo methods, collocation or other projection methods. In this talk we deal with the stochastic Galerkin method. We discuss some main issues of the discretization, such as types of approximation spaces, Karhunen-Loeve expansion of coefficients of the equation, and structures of resulting system matrices. We also show some preconditioning methods.

Numerical methods for the solution of the observer problem for nonlinear systems

Branislav Reháč

Observer problem is one of key problems of the control theory. A solution of this problem for nonlinear systems relies upon the solution of a certain partial differential equation of first order. Three methods for the numerical solution of this problem will be presented and compared. One is based on the Taylor expansions, the other one uses iterative approximations of a certain stable manifold and the third one uses finite element method.

Goal-oriented error estimates and where to find them

Vít Dolejší, Filip Roskovec

Like fantastic beasts, the errors in numerical computations arise from unforeseen places threatening to ruin all the well-meant efforts to provide a trustworthy numerical solution. The goal-oriented error estimation method tries to be smart and not to suffer a defeat in a unequal fight. It gives up measuring the inaccuracy of the numerical solution itself but rather tries to control the error of some solution-dependent quantity of interest with all its might. This union of efforts works surprisingly well in many cases keeping the errors on a tight rein in a very efficient manner.

The method is introduced both for linear and nonlinear PDEs and its strong and weak spots are discussed. We focus on several aspects of the method, namely, higher order reconstructions, adjoint consistency of the discretizations, control of the errors arising from iterative solutions of both primal and adjoint algebraic systems, and linking the estimates with the hp -anisotropic mesh adaptation.

Alternativní optimalizační úlohy pro konstrukci lisovacích nástrojů při tvarování skleněných výrobků

Petr Salač

V příspěvku je představeno pět úloh tvarové optimalizace konstrukce razníku pro lisování skleněné produkce. Minimalizace účelového funkcionálu ve tvaru druhé mocniny rozdílu povrchové teploty razníku od předem zvolené konstanty ve váhovém prostoru je prováděna změnou tvaru vnitřní dutiny razníku, změnou rychlosti chladicí proudící vody vynucené regulačním proudovým tělesem, změnou vodivosti stěny razníku způsobenou vložením izolační bariéry, změnou množství lokálně dodávané chladicí vody a změnou tvaru pevného jádra s vysokou tepelnou vodivostí.

V každé z optimalizačních úloh je prezentována stavová úloha a porovnána efektivita optimalizace na modelové numerické úloze.

Some ways of multivariate data approximation using radial basis functions

Karel Segeth

A general way of data approximation called the smooth approximation was introduced by Talmi and Gilat in 1977. Its generality consists in the possibility to minimize the sum of L_2 norms of some chosen partial derivatives of the approximant in the domain of approximation. The solution of this minimum problem provides an approximant with smooth derivatives chosen.

Some approximants obtained in this way employ radial basis functions (RBF). We compare several other types of RBF approximants and show their mutual relations. The results hold for the data interpolation, too.

Efficient multidimensional data approximation is important for solving practical problems in many branches of science and engineering. A 1D numerical example is presented.

Solvability of the power flow problem in DC overhead wire circuit modelling

Jakub Ševčík, Jan Přikryl, Lukáš Adam, Václav Šmídl

For a proper traffic simulation of electric vehicles, that draw energy from overhead wires (trolleybuses, trams, trains), modelling of the traction infrastructure is highly desirable. Since the vehicles' power demands are dependent on the traffic situation, solving overhead wire DC electrical circuit and the associated non-linear power flow problem is necessary.

Although the Newton-Raphson method is well-known and widely accepted for seeking solution of the power flow problem, the existence of the solution itself is not guaranteed. Particularly in cases where the vehicle's power demand is too high (during accelerating), the solution of the studied problem may not exist. For that reason, we introduce a scaling parameter that ensures the existence of the solution. Moreover, an iterative algorithm to find the optimal value of the proposed scaling parameter is suggested.

The algorithm is compared with standard Matlab's non-linear programming solver and the performance of the algorithm is demonstrated using own implementation into the traffic micro-simulator SUMO, a popular open-source traffic simulation platform.

Lanczosova metoda v konečné aritmetice

Dorota Šimonová

Príspevek se zaměřuje na chování Lanczosova algoritmu v konečné aritmetice. Součástí je připomenutí samotného algoritmu a jeho základních vlastností. Jelikož je chování algoritmu silně ovlivněno konečnou aritmetikou, některé vlastnosti algoritmu se ztrácejí už po pár krocích. Shrňme dosavadní výsledky analýzy chování Lanczosova algoritmu v konečné aritmetice a zformulujeme současné problémy této analýzy.

Towards a parallel domain decomposition solver for immersed boundary finite element method

Jakub Šístek

First, we recall the algorithm of the multilevel balancing domain decomposition based on constraints (BDDC), and discuss its combination with the finite element method (FEM) using an adaptive mesh refinement (AMR). Then, we focus on the extension of the solver to large-scale simulations using immersed boundary finite element method (FEM). Immersed boundary FEM presents an attractive approach to simulations avoiding the generation of body-fitted meshes, which can be tedious and challenging for complex geometries. However, the price to pay are more complicated methods for the weak enforcement of Dirichlet boundary conditions, poor conditioning of the stiffness matrices, and nonstandard numerical integration at the vicinity of the boundary. We present the concepts, our implementation, and preliminary numerical results for the Poisson problem. This is a joint work with Fehmi Cirak, Eky Febrianto, and Pavel Kůs.

The multiplicative Schwarz method for matrices with a special block structure

Petr Tichý

We analyze the convergence of the (algebraic) multiplicative Schwarz method applied to linear algebraic systems with matrices having a special block structure that arises, for example, when a (partial) differential equation is posed and discretized on a domain that consists of two subdomains with an overlap. This is a basic situation in the context of domain decomposition methods. Our analysis is based on the algebraic structure of the Schwarz iteration matrices, and we derive error bounds that are based on the block diagonal dominance of the given system matrix. Our analysis does not assume that the system matrix is symmetric (positive definite), or has the M - or H-matrix property. Our approach significantly generalizes an analysis for a special one-dimensional model problem published in our previous paper.

Numerical solution of traffic flow model on networks using numerical fluxes at junctions

Lukáš Vacek

We describe the simulation of traffic flow on networks. On individual roads we use standard macroscopic traffic models. The discontinuous Galerkin method in space and a multistep method in time is used for the numerical solution. We introduce limiters to keep the density in an admissible interval as well as prevent spurious oscillations in the numerical solution. To solve traffic networks, we construct suitable numerical fluxes at junctions. Numerical experiments are presented.

Výpočtové modelování šíření trhlin v kvazikřehkých kompozitních materiálech

J. Vala, V. Kozák

Obtížnost výpočtového modelování šíření trhlin v kvazikřehkých kompozitních materiálech, jakými jsou ve stavebnictví materiály s cementovou maticí a výtuznými vlákny rozličného původu, tkví v nutnosti zohlednění i) formování a šíření materiálových zón oslabených mikrotrhlinami a ii) vzniku a rozvoje soustavy makrotrhlin vlivem zatížení konstrukce. V tomto příspěvku poukážeme na možnost deterministické výpočtové predikce i) a ii) s využitím obecně nelokálního viskoelastického materiálového modelu a kohezních kontaktů, metody časové diskretizace a rozšířené metody konečných prvků včetně ukázkového příkladu. Naznačíme též směry dalších potřebných zobecnění.

Mathematical modelling of healthy vocal folds vibration excited by fluid flow

J. Valášek, P. Sváček, J. Horáček

The contribution deals with the fluid-structure interaction (FSI) simulation of the human vocal folds model excited by the fluid flow. The special attention is paid to the modelling of the channel closure, which is one of the main attribute of the healthy voice. The FSI mathematical model consists of the linear elasticity model (vocal fold) and the fluid flow described by the viscous incompressible Navier-Stokes equations. The arbitrary Lagrangian-Euler method (ALE) is employed for purpose of including the effect of time varying computational domain on the fluid flow computation. A few techniques how channel closing can be treated in numerical simulations are discussed. These techniques are typically based on special choice of boundary conditions, ALE deformation algorithms, remeshing procedures, etc. The whole coupled problem is solved by an in-house finite element method solver. The influence of the channel gap decreasing on the character of FSI results will be discussed. The extension of this FSI problem to the fluid-structure-acoustic interaction problem will be addressed, where the role of acoustic sources (aerodynamical or vibration-borne) will be discussed.

Viskózní materiálové modely v dynamické analýze konstrukcí

M. Trcala, I. Němec, F. Hokeš, A. Vaněčková

Článek se zabývá analýzou dynamického chování betonového konstrukčního prvku při rychlých dynamických dějích. Pro popis chování materiálu je zapotřebí zvolit vhodný konstitutivní model, který bude zohledňovat i materiálovou viskozitu. V této analýze je třeba použít složitější viskózní materiálové modely, které dokáží postihnout např. tlumení kmitání a závislost pevnosti nebo i celých pracovních diagramů na rychlosti přetvoření. Tyto složitější modely jsou často tvořeny kombinacemi visko-elastických modelů s visko-plastickými modely nebo se zviskózněnými modely poškození, pro které jsou provedeny numerické simulace. Numerická analýza je podpořena experimentálním měřením.

Guaranteed error bounds for eigenfunctions

Xuefeng Liu, Tomáš Vejchodský

We propose a new fully computable a posteriori error estimator for eigenfunctions of compact self-adjoint operators in Hilbert spaces. The estimator provides guaranteed upper bound on the approximation error, applies well to the case of tight clusters and multiple eigenvalues and is easily computable in terms of approximate eigenfunctions themselves and two-sided bounds of eigenvalues. We will present numerical examples illustrating the efficiency of this approach for the Laplace operator in both convex and non-convex domains.

Save our stones – hysteresis phenomenon in porous media flow

J. Lamač, M. Vlasák

We deal with a coupled problem of carbonation of hydrated lime and mass transport in porous media. The solution of this problem can be then applied in the context of preserving historical stone monuments. We derive a corresponding mathematical model for this problem with an important role played by the hysteresis phenomenon. We present numerical experiments for simplified model problem.

Equivalence of local- and global-best approximations (a posteriori tools in a priori analysis)

Martin Vohralík

Potential and flux reconstructions are nowadays well-established procedures in a posteriori error analysis. We show that they also lead to simple a priori error estimates as well, in that they yield equivalences of local-best and global-best approximations. More precisely, let a H^1 function be given. Then its (global-)best approximation in the H^1 seminorm by continuous piecewise polynomials has, up to a constant, the same precision as its (local-)best approximation by (discontinuous) piecewise polynomials. This yields optimal a priori error estimates for conforming finite elements under minimal regularity. Similarly, given an arbitrary function in $H(\text{div})$, we show that the error attained by its (global-)best approximation by $H(\text{div})$ -conforming piecewise polynomials, under additional constraints on the divergence and normal flux on the boundary, is equivalent to the sum of independent (local-)best approximation errors over individual mesh elements, without any constraints on the divergence or normal fluxes. This yields optimal a priori error estimates for mixed finite elements under minimal regularity.

Selected recent outcomes in the theory of core problems in tensor settings

Iveta Hnětynková, Martin Plešinger, Jana Žáková

Consider a linear approximation problem $Ax \approx b$, $b \notin \text{range}(A)$, which we want to solve in the so-called total least squares (TLS) sense. Since this problem may not have a solution, the *core problem* theory was developed.

The matrix variant $AX \approx B$, where $B \in \mathbb{R}^{m \times d}$, motivated a generalization of the core problem, which, contrary to the vector case, still may not have a TLS solution. In order to better understand the matrix case and to see it in a wider context, we show how to extend the concepts with the use of tensors into:

- Tensor right-hand side problems $A \times_1 \mathcal{X} \approx \mathcal{B}$.
- Problems with generalized models, in particular $A_{\mathcal{L}} X A_{\mathcal{R}} \approx B$.

We point out similarities and differences between tensor based formulations and their matricized counter-parts, focusing especially on the differences in core problems therein. We also propose ways to use Krylov subspace methods for extraction of core problems.